CIVL 7/8012
In-class solutions
Hypothesis testing

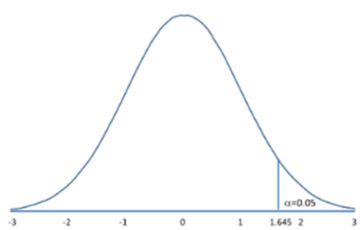
Note:

The acceptance or rejection of the null hypothesis is based on the type of test (lower or upper tailed test) which determines the rejection region.

For example, for α=0.05, when using z stat. The rejection region is shown in the figure below.
For upper-tailed test: Rejection region is $z_{cal} \geq 1.645$
For lower-tailed test: Rejection region is $z_{cal} \leq -1.645$
For two-tailed test: Rejection region is $z_{cal} \leq -1.96$ and $z_{cal} \geq 1.96$

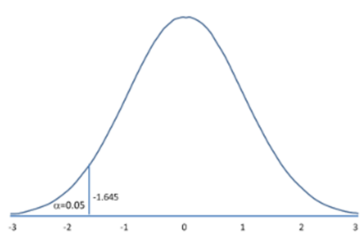
Regardless of the sign of z_{cal} (positive or negative), if these conditions are satisfied, we reject the null hypothesis.

Similarly, for comparison of means testing can be done considering the following:
$\mu_1 - \mu_2 > \Delta$ as upper-tailed test
$\mu_1 - \mu_2 < \Delta$ as lower-tailed test



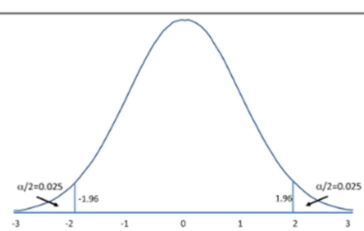
Rejection Region for Upper-Tailed Z Test (H_1: $\mu > \mu_0$) with α=0.05
The decision rule is: Reject H_0 if $Z \geq 1.645$.

Upper-Tailed Test	
α	Z
0.10	1.282
0.05	1.645
0.025	1.960
0.010	2.326
0.005	2.576
0.001	3.090
0.0001	3.719



Rejection Region for Lower-Tailed Z Test (H_1: $\mu < \mu_0$) with α =0.05
The decision rule is: Reject H_0 if $Z \leq -1.645$.

Lower-Tailed Test	
a	Z
0.10	-1.282
0.05	-1.645
0.025	-1.960
0.010	-2.326
0.005	-2.576
0.001	-3.090
0.0001	-3.719



Rejection Region for Two-Tailed Z Test (H_1: $\mu \neq \mu_0$) with α =0.05
The decision rule is: Reject H_0 if $Z \leq -1.960$ or if $Z \geq 1.960$.

Two-Tailed Test	
α	Z
0.20	1.282
0.10	1.645
0.05	1.960
0.010	2.576
0.001	3.291
0.0001	3.819

1.

i. Defining the null hypothesis, H$_o$: $\mu = \mu_0$

ii. Developing the alternative hypothesis, H$_1$: $\mu < \mu_0$

Evaluating the test statistic:

$\bar{x} = 40{,}758$

$\mu_0 = 40{,}000$

$\sigma = 1500$

$n = 16$

$$z_{calc} = \frac{40{,}758 - 40{,}000}{1500/\sqrt{16}} = 2.012$$

iii. Defining the rejection region:

It's a lower tailed test.

$z_{tab} = z_{0.01} = -2.33$

The rejection region is $z_{tab} > z_{calc}$

iv. Making a conclusion:

$z_{tab} < z_{calc}$, therefore, fail to reject the null hypothesis

2.

i. Defining the null hypothesis, H$_o$: $\mu = \mu_0$

ii. Developing the alternative hypothesis, H$_1$: $\mu > \mu_0$

iii. Evaluating the test statistic:

$\bar{x} = 3{,}109\ psi$

$\mu_0 = 3{,}200\ psi$

$s = 156\ psi$

$n = 36$

$$z_{calc} = \frac{3{,}109 - 3{,}200}{156/\sqrt{36}} = -3.5$$

iv. Defining the rejection region:

It's an upper tailed test.

$z_{tab} = z_{0.05} = 1.645$

So, the rejection region is $z_{tab} < z_{calc}$

v. Making a conclusion:

$z_{tab} > z_{calc}$, therefore, fail to reject the null hypothesis

3.

i. Defining the null hypothesis, H$_o$: $\mu = \mu_0$

ii. Developing the alternative hypothesis, H$_1$: $\mu > \mu_0$

iii. Evaluating the test statistic:

$\bar{x} = 30.316\ psi$

$\mu_0 = 30$

$s = 1.319$

$n = 6$

$$z_{calc} = \frac{30.316 - 30}{1.319/\sqrt{6}} = 0.586$$

iv. Defining the rejection region:
It's an upper tailed test.
$t_{tab} = t_{0.05,5} = 2.015$
So, the rejection region is $t_{tab} < t_{calc}$

v. Making a conclusion:
$t_{tab} > t_{calc}$, therefore, fail to reject the null hypothesis

4.

i. Defining the null hypothesis, H$_o$: $\mu_1 - \mu_2 = \Delta_0 = 0$

ii. Developing the alternative hypothesis, H$_1$: $\Delta_0 \neq 0$

iii. Evaluating the test statistic:
$\overline{x_1} = 29.8$
$\overline{x_2} = 34.7$
$\sigma_1 = 4$
$\sigma_2 = 5$
$n_1 = 20$
$n_2 = 25$

$$z_{calc} = \frac{29.8 - 34.7 - 0}{\sqrt{\frac{4^2}{20} + \frac{5^2}{25}}} = -3.65$$

iv. Defining the rejection region:
It's a two tailed test.
$z_{tab} = z_{0.005} = 2.575$
So, the rejection region is $z_{tab} < |z_{calc}|$

v. Making a conclusion:
$z_{tab} < |z_{calc}|$ therefore, reject the null hypothesis

5.

i. Defining the null hypothesis, H$_o$: $\mu_1 - \mu_2 = \Delta_0 = 0$

ii. Developing the alternative hypothesis, H$_1$: $\mu_1 - \mu_2 > 0$

iii. Evaluating the test statistic:
$\overline{x_1} = 9.9$
$\overline{x_2} = 16.7$
$s_1 = 4.9$
$s_2 = 7$
$n_1 = 30$
$n_2 = 35$

$$z_{calc} = \frac{9.9 - 16.7 - 0}{\sqrt{\frac{4.9^2}{30} + \frac{7^2}{35}}} = -4.585$$

iv. Defining the rejection region:
It's an upper tailed test.

$z_{tab} = z_{0.01} = 2.33$

So, the rejection region is $z_{tab} < z_{calc}$

v. Making a conclusion:

$z_{tab} > z_{calc}$, therefore, fail to reject the null hypothesis

6.

i. Defining the null hypothesis, H₀: $\mu_1 - \mu_2 = \Delta_0 = 5$

ii. Developing the alternative hypothesis, H₁: $\mu_1 - \mu_2 > 5$

iii. Evaluating the test statistic:

$\overline{x_1} = 110\ kV$

$\overline{x_2} = 101\ kV$

$s_1 = 24\ kV$

$s_2 = 22\ kV$

$n_1 = 15$

$n_2 = 76$

$\Delta_0 = 5$

$$s_p = \sqrt{\frac{(15-1)*24^2 + (76-1)*22^2}{15+76-2}} = 22.32$$

$$t_{calc} = \frac{110 - 101 - 5}{22.32 * \sqrt{\frac{1}{15} + \frac{1}{76}}} = 0.634$$

iv. Defining the rejection region:

It's an upper tailed test.

$t_{tab} = t_{0.01,89} = 2.37$

The rejection region is $t_{tab} < t_{calc}$

v. Making a conclusion:

$t_{tab} > t_{calc}$, therefore, fail to reject the null hypothesis

7.

i. Defining the null hypothesis, H₀: $\mu_1 - \mu_2 = \Delta_0$

ii. Developing the alternative hypothesis, H₁: $\mu_1 - \mu_2 > 0$

iii. Evaluating the test statistic:

$\overline{x_1} = 22.36$

$\overline{x_2} = 13.78$

$s_1^2 = 28.63$

$s_2^2 = 1.8$

$n_1 = n_2 = 5$

$$v = \frac{\left(\frac{28.63}{5} + \frac{1.8}{5}\right)^2}{\left(\frac{\left(28.63/5\right)^2}{4} + \frac{\left(1.8/5\right)^2}{4}\right)} = \frac{37.039}{8.19 + 0.0324} = 4.5$$

$$t_{calc} = \frac{22.36 - 13.78 - 0}{\sqrt{\frac{28.63}{5} + \frac{1.8}{5}}} = 3.477$$

iv. Defining the rejection region:
It's an upper tailed test.
$t_{tab} = t_{0.01,4} = 3.747$
The rejection region is $t_{tab} < t_{calc}$

v. Making a conclusion:
$t_{tab} > t_{calc}$ Fail to Reject the null hypothesis

8.

i. Defining the null hypothesis, H$_o$: $\mu_D = 0$

ii. Developing the alternative hypothesis, H$_1$: $\mu_D > 0$

iii. Evaluating the test statistic:

Treated	Untreated	d_i	$\bar{d}$	$(d_i-\bar{d})^2$	S_D^2
16.1	14.8	1.3	1.42	0.0144	0.107
14.7	13.2	1.5		0.0064	
17.4	15.5	1.9		0.2304	
13.7	12.3	1.4		0.0004	
16.9	15.9	1		0.1764	

$$t_{paired} = \frac{1.42 - 0}{0.327/\sqrt{5}} = 9.71$$

iv. Defining the rejection region:
It's an upper tailed test.
$t_{tab} = t_{0.01,4} = 3.747$
The rejection region is $t_{tab} < t_{paired}$

v. Making a conclusion:
$t_{tab} < t_{paired}$ Reject the null hypothesis

9.

i. Defining the null hypothesis, H$_o$: $\sigma^2 = \sigma_0^2$

ii. Developing the alternative hypothesis, H$_1$: $\sigma^2 < \sigma_0^2$

iii. Evaluating the test statistic:
$s^2 = 0.0153\ oz^2$
$\sigma_0^2 = 0.01\ oz^2$
$n = 20$
$$\chi^2_{calc} = \frac{(20-1) * 0.0153}{0.01} = 29.07$$

iv. Defining the rejection region:
It's a lower tailed test.
$\chi^2_{tab} = \chi^2_{0.95,19} = 10.12$
The rejection region is $\chi^2_{tab} > \chi^2_{calc}$

v. Making a conclusion:

$\chi^2{}_{tab} < \chi^2{}_{calc}$ Fail to reject the null hypothesis

10.

Arranging given data to fit the subscripts $s_1^2 > s_2^2$

Material 1	Material 2
$n_1 = 16$	$n_2 = 25$
$x_1 = 370\ lb$	$x_2 = 380\ lb$
$s_1^2 = 400$	$s_2^2 = 100$

i. Defining the null hypothesis, H_o: $\sigma_1^2 = \sigma_2^2$

ii. Developing the alternative hypothesis, H_1: $\sigma_1^2 \neq \sigma_2^2$

iii. Evaluating the test statistic:

$$F_{calc} = \frac{400}{100} = 4$$

$v1 = 15, v2 = 24$

iv. Defining the rejection region:

It's a two tailed test.

$F_{tab} = F_{0.025,15,24} = 2.44$

The rejection region is $F_{tab} < F_{calc}$

v. Making a conclusion:

$F_{tab} < F_{calc}$ therefore, reject the null hypothesis

11.

i. Defining the null hypothesis, H_o: $\mu = \mu_0$

ii. Developing the alternative hypothesis, H_1: $\mu < \mu_0$

iii. Evaluating the test statistic:

$\bar{x} = 3{,}109\ psi$

$\mu_0 = 3{,}200\ psi$

$\sigma = 156\ psi$

$$z_{calc} = \frac{3109 - 3200}{156/6} = -3.5$$

$P = F(z_{calc}) = 0.00023$

iv. Defining the rejection region:

It's a lower tailed test.

P < 0.05 is the rejection region

v. Making a conclusion:

$P < 0.05$ therefore, reject the null hypothesis